Effective theory of fluctuating orbital currents in high-T_c cuprate superconductors

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We derive an effective dissipative quantum field theory for fluctuating orbital currents in clean CuO_2 sheets of high-T_c cuprates based on a three-band model. The Coulomb repulsion term between Cu and O sites is decoupled in terms of current operators representing horizontal and vertical parts of circulating currents within each CuO_2 unit cell of the lattice. The model has ordering of currents at finite temperatures. The dissipative kernel in the model is of the form $|\omega|/|\mathbf{q}|$, indicating Landau damping. Applications of the effective theory to other models are also discussed.

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Constructing an effective description of the long-wavelength and low-energy physics of high-T_c superconducting cuprates represents a profound and formidable problem in physics. Such a description must be consistent with experimental observations of several anomalous normal state properties of these systems. Varma has recently proposed that quantum critical fluctuations associated with the breakup of a subtle order, involving circulating currents, could induce the observed anomalous normal state properties of high-T_c superconductors.[1] Essentially, the associated quantum critical fluctuations are suggested to produce a fluctuation spectrum resulting in a marginal Fermi liquid.[2] Recently, such a spectrum has been derived from a conjectured effective field theory of circulating currents.[3] It should, however, be mentioned that a recent numerical evaluation of the current-current correlations in a three-band *t-J* model with 24 sites, where doubly occupied sites have been projected out, shows no evidence of the orbital current pattern.[4]

The particular form of proposed order involves circulating currents within a CuO_2 unit cell where the currents run horizontally and vertically through a Cu site and close by *direct hopping between* O *orbitals*, as in Fig. 1. Three other equivalent patterns may be found by reversing the direction of the current through each Cu site in the horizontal and vertical directions.

This results in a pattern of *staggered* orbital magnetic moments within each unit cell, such that the pattern repeats from unit cell to unit cell. A magnetic intensity of the type associated with the above orbital magnetism has recently been reported.[5] Since no obvious thermodynamic singularities have so far been reported at the pseudogap line in the cuprates, it is important to investigate whether or not the proposed models for this novel type of order imply the presence or absence of prominent signals in such quantities as specific heat or (indirectly) magnetization. Other staggered orbital magnetic patterns have also been proposed, most notably the extension of the staggered flux phase[6] to finite doping.[7] We emphasize that our justification for focusing on the orbital current pattern proposed by Varma[1] are the experiments reported in Ref. 5.

We derive an effective quantum field theory for fluctuating orbital currents from a microscopic description of clean CuO_2 planes. We are primarily interested in investigating the intrinsic effects such fluctuations have on the physics of the cuprates. We therefore neglect disorder, as was also done in Ref. 3. With ever improving sample quality, we expect that the effective theory we derive should be useful. The starting point is the three-band model

$$H = \sum_{\mathbf{r},\sigma} \varepsilon_d d^\dagger_{\mathbf{r},\sigma} d_{\mathbf{r},\sigma} + K_{pd} + K_{pp} + H^{(1)}_{\mathrm{int}} + H^{(2)}_{\mathrm{int}},$$

where

$$K_{pd} = t_{pd} \sum_{\mathbf{r},\sigma} [d^\dagger_{\mathbf{r},\sigma}(p_{x,\mathbf{r}+(a/2)\hat{\mathbf{x}},\sigma} - p_{x,\mathbf{r}-(a/2)\hat{\mathbf{x}},\sigma} - p_{y,\mathbf{r}+(a/2)\hat{\mathbf{y}},\sigma} + p_{y,\mathbf{r}-(a/2)\hat{\mathbf{y}},\sigma}) + \mathrm{H.c.}],$$

$$K_{pp} = -t_{pp} \sum_{\mathbf{r},\sigma} [(p^\dagger_{x,\mathbf{r}+(a/2)\hat{\mathbf{x}},\sigma} - p^\dagger_{x,\mathbf{r}-(a/2)\hat{\mathbf{x}},\sigma})(p_{y,\mathbf{r}+(a/2)\hat{\mathbf{y}},\sigma} - p_{y,\mathbf{r}-(a/2)\hat{\mathbf{y}},\sigma}) + \mathrm{H.c.}],$$

and

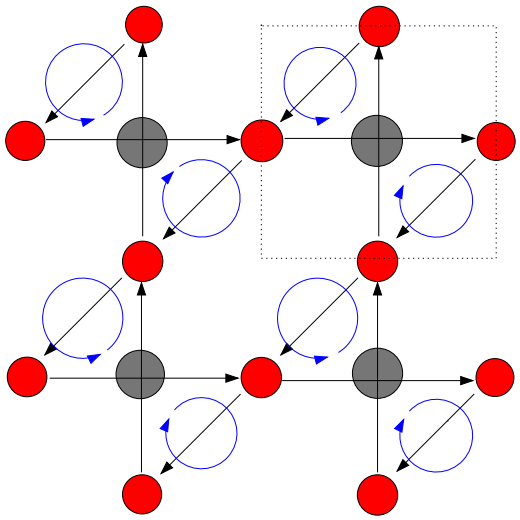

FIG. 1. (Color online) The circulating current phase Θ_{II} (Ref. 1). Cu sites are gray circles and O sites are red. The unit cell is shown by the dashed square. A staggered magnetic moment pattern within each unit cell that repeats from unit cell to unit cell (the curl of the blue directed circles) is indicated.

$$H_{\rm int}^{(2)} = V\sum_{\mathbf{r},\sigma,\sigma'} n_{d,\mathbf{r},\sigma}(n_{p_x,\mathbf{r}+(a/2)\hat{\mathbf{x}},\sigma'} + n_{p_x,\mathbf{r}-(a/2)\hat{\mathbf{x}},\sigma'} + n_{p_y,\mathbf{r}+(a/2)\hat{\mathbf{y}},\sigma'} + n_{p_y,\mathbf{r}-(a/2)\hat{\mathbf{y}},\sigma'}).$$

We work with electron operators and the vacuum is defined as empty $d_{x^2-y^2}$, p_x, and p_y orbitals. The $\mathbf{r}$ sum runs over the Cu lattice. The Cu-O and O-O hopping is governed by the parameters t_{pd} and t_{pp}, respectively, whereas ε_d is the difference in on-site energy between the copper and oxygen orbitals. The term $H_{\rm int}^{(1)}$ represents on-site repulsion terms, for which we make the crude assumption that their effect is to merely renormalize the hopping parameters $t_{pd}\to\bar{t}_{pd}=|x|t_{pd}$, $t_{pp}\to\bar{t}_{pp}=|x|t_{pp}$, where $|x|$ is the deviation from half-filling.[1] We also assume the O-O repulsion to be small. Hence, we only consider explicitly $H_{\rm int}^{(2)}$, the Cu-O repulsion.

The interaction term $H_{\rm int}^{(2)}$ can be decoupled[8] in terms of bosonic fields coupling to the bilinear fermion operators $A^{(i)}_{\mathbf{q},\sigma,\sigma'} \equiv N^{-1/2}\Sigma_{\mathbf{k}}(a^{(i)}_{x,\mathbf{k}-\mathbf{q}}p^{\dagger}_{x,\mathbf{k}-\mathbf{q},\sigma'} + a^{(i)}_{y,\mathbf{k}-\mathbf{q}}p^{\dagger}_{y,\mathbf{k}-\mathbf{q},\sigma'})d_{\mathbf{k},\sigma}$ with $i=1,\ldots,4$.[1] Here, N is the number of Cu lattice sites. We define $a^{(1)}_{x,\mathbf{k}}=a^{(2)}_{x,\mathbf{k}}=\sin(k_x a/2)\equiv s_{x,k}$, $a^{(3)}_{x,\mathbf{k}}=a^{(4)}_{x,\mathbf{k}}=\cos(k_x a/2)\equiv c_{x,k}$ and $a^{(1)}_{y,\mathbf{k}}=-a^{(2)}_{y,\mathbf{k}}=s_{y,k}$, $a^{(3)}_{y,\mathbf{k}}=-a^{(4)}_{y,\mathbf{k}}=c_{y,k}$, where a is the Cu-Cu lattice constant. A discussion of $\langle A^{(i)}_{0,\sigma,\sigma'}\rangle$ as translational invariant order parameters in the cuprates is found in Ref. 1 While $i=2$ transforms as the kinetic energy, $i=1$ and $i=3,4$ *give rise to different current patterns*. Since the observed magnetic signal[5] is consistent with the current patterns of $i=3,4$, we keep only this in what follows. An effective model for the $i=1$ part was considered in Ref. 9 Observe the relation $N^{-1/2}\Sigma_{\mathbf{k}}c_{x,k-q}p^{\dagger}_{x,\mathbf{k}-\mathbf{q},\sigma'}d_{\mathbf{k},\sigma}=1/4(\kappa^{x}_{\mathbf{q},\sigma,\sigma'}+ij^{x}_{\mathbf{q},\sigma,\sigma'})$, where, in real space,

$$j^{x}_{\mathbf{r},\sigma,\sigma'} \equiv \frac{i}{2}[d^{\dagger}_{\mathbf{r},\sigma}(p_{x,\mathbf{r}+(a/2)\hat{\mathbf{x}},\sigma'} + p_{x,\mathbf{r}-(a/2)\hat{\mathbf{x}},\sigma'}) - \text{H.c.}]. \quad (1)$$

In a unit cell centered on Cu, this is proportional to the current from the left oxygen to the copper *plus* the current from copper to the right oxygen. We define $j^{y}_{\mathbf{r},\sigma,\sigma'}$ in the same way but with a minus sign due to the d-wave symmetry of the Cu orbital. Finite expectation values of $\kappa^{x(y)}_{\mathbf{r},\sigma,\sigma'}$ would correspond to Landau-Pomeranchuk instabilites, believed not to be relevant in the cuprates. Thus, we retain only the decoupling fields that correspond to spin diagonal expectation values of the operators $j^{x(y)}_{\mathbf{r},\sigma,\sigma'}(\tau)$ since $\langle j^{x(y)}_{\mathbf{r},\sigma,\sigma}(\tau)\rangle \neq 0$ in the current pattern depicted in Fig. 1. The fields retained, $J^{x(y)}_{\mathbf{r}}(\tau)$, are real and $\langle J^{x(y)}_{\mathbf{r}}(\tau)\rangle = V\langle j^{x(y)}_{\mathbf{r},\sigma,\sigma'}(\tau)\rangle\delta_{\sigma,\sigma'}$, i.e., the fields represent charge currents on horizontal and vertical O-Cu-O links. The fields $J^{x(y)}_{\mathbf{r}}(\tau)$ and the fermions are coupled by particle-hole excitations of the form $i\Sigma_{\mathbf{k},\mathbf{q},\sigma}[J^{x}_{-\mathbf{q}}c_{x,k-q}p^{\dagger}_{x,\mathbf{k}-\mathbf{q},\sigma}d_{\mathbf{k},\sigma} - (x\to y) - \text{H.c.}]$, where the time dependence was omitted. It is important to keep in mind that the bosonic fields $J^{x(y)}_{\mathbf{r}}(\tau)$ transform as vectors under a change of coordinate system. Note that we could also have chosen the arguments of the $a^{(i)}$'s to be $\mathbf{k}$ and not $\mathbf{k}-q$ in $A^{(i)}_{\mathbf{q},\sigma,\sigma'}$, corresponding to a decoupling in terms of currents defined on horizontal and vertical Cu-O-Cu links.

Integrating out the fermion fields, we obtain the partition function as $Z=\int DJ^{x}DJ^{y}e^{-S}$, where the effective action is given by $S=\frac{1}{2V}\Sigma_{\mathbf{q},\omega_\nu}[J^{x}_{\mathbf{q}}(i\omega_\nu)J^{x}_{-\mathbf{q}}(-i\omega_\nu)+J^{y}_{\mathbf{q}}(i\omega_\nu)J^{y}_{-\mathbf{q}}(-i\omega_\nu)] - \text{Tr}\ln[\mathcal{G}_0^{-1}+\Sigma]$. Using the gauge transformation $p_{x,\mathbf{k},\sigma}\to ip_{x,\mathbf{k},\sigma}, p_{y,\mathbf{k},\sigma}\to -ip_{y,\mathbf{k},\sigma}$, we have

$$\mathcal{G}^{-1}_{0,\mathbf{k}_1\mathbf{k}_2,\sigma_1\sigma_2}(i\omega_{n_1},i\omega_{n_2}) = \delta_{\mathbf{k}_1,\mathbf{k}_2}\delta_{n_1,n_2}\delta_{\sigma_1,\sigma_2} \times \begin{pmatrix} -i\omega_{n_1}+\varepsilon_d-\mu & 2t_{pd}s_{x,k_1} & 2t_{pd}s_{y,k_1} \\ 2t_{pd}s_{x,k_1} & -i\omega_{n_1}-\mu & 4t_{pp}s_{x,k_1}s_{y,k_1} \\ 2t_{pd}s_{y,k_1} & 4t_{pp}s_{x,k_1}s_{y,k_1} & -i\omega_{n_1}-\mu \end{pmatrix}, \quad (2)$$

$$\Sigma_{\mathbf{k}_1\mathbf{k}_2,\sigma_1\sigma_2}(i\omega_{n_1},i\omega_{n_2}) = \frac{\delta_{\sigma_1,\sigma_2}}{\sqrt{\beta N}} \times \begin{pmatrix} 0 & c_{x,k_2}J^{x}_{\mathbf{k}_{12}}(i\omega_{12}) & c_{y,k_2}J^{y}_{\mathbf{k}_{12}}(i\omega_{12}) \\ c_{x,k_1}J^{x}_{\mathbf{k}_{12}}(i\omega_{12}) & 0 & 0 \\ c_{y,k_1}J^{y}_{\mathbf{k}_{12}}(i\omega_{12}) & 0 & 0 \end{pmatrix}, \quad (3)$$

where we have defined $\mathbf{k}_{12}\equiv\mathbf{k}_1-\mathbf{k}_2$ and $\omega_{12}\equiv\omega_{n_1}-\omega_{n_2}$. For $t_{pp}=0$, the noninteracting part of the problem $\mathcal{G}_0^{-1}$ may easily be diagonalized into three quasiparticle bands $E^{(0)}_{\mathbf{k}}=0, E^{(\pm)}_{\mathbf{k}}=\varepsilon_d/2\pm\sqrt{(\varepsilon_d/2)^2+4t_{pd}^2(s^2_{x,k}+s^2_{y,k})}$, of which $E^{(0)}_{\mathbf{k}}, E^{(-)}_{\mathbf{k}}$ are full and $E^{(+)}_{\mathbf{k}}$ is partially filled. This picture is not qualitatively altered by $t_{pp}\neq 0$. A nonzero value of t_{pp} is, however, vital for the realization of the current pattern. It is implicit that $\langle J^{x(y)}_{\mathbf{r}}(\tau)\rangle\to 0$ when $t_{pp}\to 0$.[1]

Expanding the last term,[10] odd powers of J vanish, such that $\text{Tr}\ln[\mathcal{G}_0^{-1}+\Sigma]=\text{Tr}\ln\mathcal{G}_0^{-1}-\frac{1}{2}\text{Tr}[\mathcal{G}_0\Sigma]^2+\mathcal{O}(J^4)$, where $-\text{Tr}\ln\mathcal{G}_0^{-1}$ gives the free energy of the noninteracting system and Σ involves the fluctuating fields J^x and J^y. To second order in the fields $J^{x(y)}$ and in space and imaginary time gradients, we have derived a quantum dissipative effective action $S=S_C+S_Q$, where

$$S_C = \sum_{\mathbf{q},\omega_\nu}\sum_{i,j=x,y} G^{-1}_{C,ij}J^{i}_{\mathbf{q}}(i\omega_\nu)J^{j}_{-\mathbf{q}}(-i\omega_\nu),$$

$$S_Q = \sum_{\mathbf{q},\omega_\nu}\sum_{i,j=x,y} G^{-1}_{Q,ij}J^{i}_{\mathbf{q}}(i\omega_\nu)J^{j}_{-\mathbf{q}}(-i\omega_\nu), \quad (4)$$

with $G^{-1}_{C,xx}=\alpha_c+\alpha_t q_x^2+\alpha_t q_y^2$, $G^{-1}_{C,yy}=\alpha_c+\alpha_l q_y^2+\alpha_t q_x^2$, $G^{-1}_{C,xy}=G^{-1}_{C,yx}=\alpha_{xy}q_xq_y$, $G^{-1}_{Q,xx}=\alpha_0\omega_\nu^2+\alpha_d\frac{|\omega_\nu|}{|\mathbf{q}|}\hat{q}_y^2$, $G^{-1}_{Q,yy}=\alpha_0\omega_\nu^2+\alpha_d\frac{|\omega_\nu|}{|\mathbf{q}|}\hat{q}_x^2$, and $G^{-1}_{Q,xy}=G^{-1}_{Q,yx}=-\alpha_d\frac{|\omega_\nu|}{|\mathbf{q}|}\hat{q}_x\hat{q}_y$. Here, $\hat{q}_x=q_x/|\mathbf{q}|$. The dissipation kernel is valid for $|\omega_\nu|/|\mathbf{q}|\ll 1$. The limit $|\omega_\nu|/|\mathbf{q}|\gg 1$ does not contribute to dissipation. The explicit expressions for the coefficients α_i are unwieldy and of limited use. The equality of the diagonal and off-diagonal dissipation coefficients is only correct when $t_{pp}=0$. Changes when $t_{pp}\neq 0$ are small and unimportant and are neglected in the following. Note also that this theory might not be appli-

cable to the ordered phase since the Fermi surface is proposed to be gapped there.[1] However, it is the fluctuation spectrum in the disordered phase which is important in connection with the marginal fermi liquid hypothesis.[2]

We have divided the action into a classical (C) and a quantum (Q) part. At finite temperatures, only the classical piece of the action S_C needs to be considered as far as critical properties are concerned. The excitation energies of the eigenmodes of S_C are given by $\lambda_{\pm}=\alpha_c+(\alpha_l+\alpha_t)q^2/2\pm\sqrt{(\Delta\alpha)^2q^4+\gamma q_x^2q_y^2}$, where $\Delta\alpha=(\alpha_l-\alpha_t)/2$ and $\gamma=\alpha_{xy}^2-(2\Delta\alpha)^2$. Hence, for $(\alpha_l,\alpha_t)>0$, a uniformly ordered state is stable in the classical domain below some critical temperature, provided that $\alpha_{xy}^2<\alpha_l^2+\alpha_t^2$.

The dissipation kernel essentially gives Landau damping, albeit anisotropic due to the directional nature of the fields. The dissipation is a result of coupling to the gapless particle-hole excitations in the band $E_{\mathbf{k}}^{(+)}$. The singular form $|\omega_\nu|/|\mathbf{q}|$ is correct only if the orders in the horizontal and vertical currents are *uniform* and not modulated at some nonzero reciprocal vector. It implies that the dynamical critical exponent $z=3$ (Ref. 12) (see however, Ref. 10).

Current amplitude fluctuations are expected to be high-energy excitations[1] and will therefore not determine the critical properties of the model. Thus, we treat the fields $J_{\mathbf{r}}^x(\tau)$ and $J_{\mathbf{r}}^y(\tau)$ as *Ising variables*. Reverting to a real space Cu lattice formulation and setting $a=1$, we obtain (up to constant terms)

$$S_C=-\int_0^\beta d\tau\Bigg\{\sum_{\langle\mathbf{r},\mathbf{r}'\rangle}[\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^x J_{\mathbf{r}}^x(\tau)J_{\mathbf{r}'}^x(\tau)+\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^y J_{\mathbf{r}}^y(\tau)J_{\mathbf{r}'}^y(\tau)]+\sum_{\langle\langle\mathbf{r},\mathbf{r}'\rangle\rangle}\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^{xy}[J_{\mathbf{r}}^x(\tau)J_{\mathbf{r}'}^y(\tau)+J_{\mathbf{r}}^y(\tau)J_{\mathbf{r}'}^x(\tau)]\Bigg\},$$

$$S_Q=\tilde{\alpha}_0\int_0^\beta d\tau\sum_{\mathbf{r}}\left[\left(\frac{\partial J_{\mathbf{r}}^x}{\partial\tau}\right)^2+\left(\frac{\partial J_{\mathbf{r}}^y}{\partial\tau}\right)^2\right]+\tilde{\alpha}_d\int_0^\beta d\tau d\tau'\sum_{\mathbf{r},\mathbf{r}'}\sum_{i,j}\times[J_{\mathbf{r}}^i(\tau)-J_{\mathbf{r}'}^i(\tau')]\mathbb{K}_{\mathbf{r}-\mathbf{r}'}^{ij}(\tau-\tau')[J_{\mathbf{r}}^j(\tau)-J_{\mathbf{r}'}^j(\tau')]. \quad (5)$$

Here, $\langle\mathbf{r},\mathbf{r}'\rangle$ and $\langle\langle\mathbf{r},\mathbf{r}'\rangle\rangle$ denote nearest-neighbor and next-nearest-neighbor summations, respectively. For $\mathbf{r}-\mathbf{r}'=\pm\hat{\mathbf{x}}$, $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^x=\tilde{\alpha}_l$ and $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^y=\tilde{\alpha}_t$, whereas when $\mathbf{r}-\mathbf{r}'=\pm\hat{\mathbf{y}}$, $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^x=\tilde{\alpha}_t$ and $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^y=\tilde{\alpha}_l$. The parameter $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^{xy}=\tilde{\alpha}_{xy}$ when $\mathbf{r}-\mathbf{r}'=\pm(x+\hat{\mathbf{y}})$ and $\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^{xy}=-\tilde{\alpha}_{xy}$ when $\mathbf{r}-\mathbf{r}'=\pm(\hat{\mathbf{x}}-\hat{\mathbf{y}})$. The coefficient $\tilde{\alpha}_d>0$ and the positive semidefinite matrix $\mathbb{K}_{\mathbf{r}-\mathbf{r}'}(\tau-\tau')=K_{\mathbf{r}-\mathbf{r}'}(\tau-\tau')\hat{\mathbf{g}}_{\mathbf{r}-\mathbf{r}'}\otimes\hat{\mathbf{g}}_{\mathbf{r}-\mathbf{r}'}$, where $\hat{\mathbf{g}}_{\mathbf{r}-r'}=(\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$ and $K_{\mathbf{r}}(\tau)=1/[|\mathbf{r}|\sin^2(\pi\tau/\beta)]$. Fluctuations $(J_{\mathbf{r}}^x\to-J_{\mathbf{r}}^x,J_{\mathbf{r}}^y\to J_{\mathbf{r}}^y)$ correspond to going from the depicted current pattern (Fig. 1) to a new one, which is obtained by a counterclockwise rotation of $\pi/2$, $(J_{\mathbf{r}}^x\to J_{\mathbf{r}}^x,J_{\mathbf{r}}^y\to-J_{\mathbf{r}}^y)$ corresponds to clockwise rotation of $\pi/2$, and $(J_{\mathbf{r}}^x\to-J_{\mathbf{r}}^x,J_{\mathbf{r}}^y\to-J_{\mathbf{r}}^y)$ to a rotation of π. It is implied that in the dissipation kernel, we must use a short-distance cutoff in $(\tau,\mathbf{r})$-space since the expressions are derived in the limit of low $(\omega,\mathbf{q})$. The tildes on the coefficients indicate that the model in Eq. (5) is regularized on a lattice and that the fields have been normalized to Ising variables. Moreover, there will be higher order (quartic) terms generated that simply involve local squares of Ising variables multiplied by some bilinear combination of Ising variables, and these will also contribute to the coefficients of the quadratic terms even before a renormalization group analysis is carried out. These terms are also taken into account by the tilde.

In general, we have $\tilde{\alpha}_l\neq\tilde{\alpha}_t$. A current living on a horizontal O-Cu-O link, $J_{\mathbf{r}}^x$, couples to $J_{\mathbf{r}\pm\hat{\mathbf{x}}}^x$ through $\tilde{\alpha}_l$ and to $J_{\mathbf{r}\pm\hat{\mathbf{y}}}^x$ through $\tilde{\alpha}_t$. As seen from Fig. 1, there is no reason for these couplings to be similar and, in fact, a detailed derivation shows that they are not.[14]

At finite temperature, we may ignore the inertial and dissipative terms, which reduce the model to a classical model of two coupled Ising fields. Such a classical model will suffice to study the breakup of the current pattern at finite temperatures, while its quantum critical version can only be accessed via the full dissipative field theory. Note also that the dissipation kernel is *nonlocal* both in imaginary time and in space. The latter distinguishes this dissipation term from the Caldeira-Leggett type of dissipation appropriate for an array of Josephson junctions.[13,15] The nonlocality in $\mathbf{r}$-space is anisotropic for the same reason as for the nearest-neighbor coupling.

Equation (5) may be rewritten on the form

$$S_C=-\int_0^\beta d\tau\Bigg\{\sum_{\langle\mathbf{r},\mathbf{r}'\rangle}[\bar{\alpha}\cos(\theta_{\mathbf{r},\tau}-\theta_{\mathbf{r}',\tau})+(\Delta\tilde{\alpha})_{\mathbf{r},\mathbf{r}'}\sin(\theta_{\mathbf{r},\tau}+\theta_{\mathbf{r}',\tau})]+2\sum_{\langle\langle\mathbf{r},\mathbf{r}'\rangle\rangle}\tilde{\alpha}_{\mathbf{r},\mathbf{r}'}^{xy}\cos(\theta_{\mathbf{r},\tau}+\theta_{\mathbf{r}',\tau})\Bigg\},$$

$$S_Q=2\tilde{\alpha}_0\int_0^\beta d\tau\sum_{\mathbf{r}}\left(\frac{\partial\theta_{\mathbf{r},\tau}}{\partial\tau}\right)^2+\tilde{\alpha}_d\int_0^\beta d\tau d\tau'\sum_{\mathbf{r},\mathbf{r}'}\sum_{i,j}[J^i(\theta_{\mathbf{r},\tau})-J^i(\theta_{\mathbf{r}',\tau'})]\mathbb{K}_{\mathbf{r}-\mathbf{r}'}^{ij}(\tau-\tau')[J^j(\theta_{\mathbf{r},\tau})-J^j(\theta_{\mathbf{r}',\tau'})], \quad (6)$$

where we have used the parametrization $\cos(\theta_{\mathbf{r},\tau})=[J_{\mathbf{r}}^x(\tau)+J_{\mathbf{r}}^y(\tau)]/2$, $\sin(\theta_{\mathbf{r},\tau})=[J_{\mathbf{r}}^x(\tau)-J_{\mathbf{r}}^y(\tau)]/2$, and $\theta_{\mathbf{r},\tau}\in(0,\pi/2,\pi,3\pi/2)$. We have defined $\bar{\alpha}=(\tilde{\alpha}_l+\tilde{\alpha}_t)$, $(\Delta\tilde{\alpha})_{\mathbf{r},\mathbf{r}'}=(\tilde{\alpha}_l-\tilde{\alpha}_t)$ for $\mathbf{r}-\mathbf{r}'=\pm\hat{\mathbf{x}}$ and $(\Delta\tilde{\alpha})_{\mathbf{r},\mathbf{r}'}=-(\tilde{\alpha}_l-\tilde{\alpha}_t)$ for $\mathbf{r}-\mathbf{r}'=\pm\hat{\mathbf{y}}$.

Equations (5) and (6) are the main results of this paper. These models describe a phase transition from a disordered bosonic state (a Fermi liquid) into a state with bosonic order in the form of ordered orbital currents.

We next proceed to discuss some qualitative aspects. Consider first this model at finite temperature, where we may use the approximation $S\approx S_C$. When $(\tilde{\alpha}_l,\tilde{\alpha}_t)>0$ and $\tilde{\alpha}_{xy}=0$, the current pattern in Fig. 1 repeats uniformly from unit cell to unit cell throughout the system in the ordered state. The specific heat has a logarithmic singularity at a critical temperature determined by the condition $\sinh(2\beta_c\tilde{\alpha}_l)\sinh(2\beta_c\tilde{\alpha}_t)=1$, where $\beta_c=1/T_c$. Anisotropy in the nearest-neighbor couplings suppresses the critical temperature and critical amplitudes and narrows the critical region but does not alter the universality class of the phase transition.[16] When $(\tilde{\alpha}_l,\tilde{\alpha}_t)=0$ and $\tilde{\alpha}_{xy}\neq0$, the ground state of the system features a striped

phase in the diagonal directions, irrespective of the sign of $\tilde{\alpha}_{xy}$. Note also that when $\mathrm{sign}(\tilde{\alpha}_l) \neq \mathrm{sign}(\tilde{\alpha}_t)$ and $\tilde{\alpha}_{xy}=0$, one obtains order with a period of twice the lattice constant.

The dissipative term in this model comes from the coupling of the bosonic current fields to particle-hole excitations in the partially filled band $E^{(+)}_{\mathbf{k}}$, i.e., an intraband transition. In the above, we defined the currents on horizontal and vertical O-Cu-O links, living on Cu sites. We could alternatively have defined the currents on Cu-Cu links, both in a three-band model and in a one-band model. This definition would be relevant to the study of *d*-density waves.[6,7] However, one would expect a different dissipation term in that case, due to the finite modulation vector of the ordered currents. Note also that the ω_ν^2-terms in S_Q in Eq. (4), equivalently the inertial terms in Eqs. (5) and (6), are of multiband origin.

The quartic terms in S_C that would emerge from the above treatment are of the type $\alpha^{ijlm}_{\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4} J^i_{\mathbf{r}_1} J^j_{\mathbf{r}_2} J^l_{\mathbf{r}_3} J^m_{\mathbf{r}_4}$. Note that for $i=j=x$, $l=m=y$, these terms include an Ashkin-Teller type of four-spin interaction, used in Ref. 3 to argue that the Ising type of singularity in specific heat would be quenched. S_C in Eqs. (5) and (6) differs from the model of Ref. 3 in several respects. However, a direct comparison is difficult, as it is not clear what physical quantites the fields in Ref. 3 represent. Firstly, the Ising-exchange coupling terms in Eqs. (5) and (6) are anisotropic, possibly highly anisotropic, due to the bond character of the Ising variables. Moreover, the term $\tilde{\alpha}^{xy}_{\mathbf{r},\mathbf{r}'} J^x_{\mathbf{r}} J^y_{\mathbf{r}'}$ in Eqs. (5) and (6) is absent in Ref. 3. While this term may be perturbatively irrelevant, it is far from clear that $\tilde{\alpha}_{xy}$ is actually small. In addition, there also seems to be a discrepancy between the dissipation kernel $|\omega_\nu|/|\mathbf{q}|$ derived here and the one employed in Ref. 3.

We expect our model to be generically useful in describing thermal and quantum critical fluctuations of directed particle-hole bond variables in fermionic lattice models.

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